
Teaching, Learning, and Human Development

Accelerating Mathematics Achievement Using Heterogeneous Grouping

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This longitudinal study examined the effects of providing an accelerated mathematics curriculum in heterogeneously grouped middle school classes in a diverse suburban school district. A quasi-experimental cohort design was used to evaluate subsequent completion of advanced high school math courses as well as academic achievement. Results showed that probability of completion of advanced math courses increased significantly and markedly in all groups, including minority students, students of low socioeconomic status, and students at all initial achievement levels. Also, the performance of initial high achievers did not differ statistically in heterogeneous classes relative to previous homogeneous grouping, and rates of participation in advanced placement calculus and test scores improved.

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The objective of the national “standards” movement is to define high standards for what students learn and then to hold students, educators, and schools accountable for reaching them (Heubert & Hauser, 1999; Natriello &

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Pallas, 1999). What students should know and be able to do in mathematics is a critical ingredient of the standards debate. National councils and commissions have agreed that all students should master a more challenging mathematics curriculum (National Council of Teachers of Mathematics, 1989; Pendergast, 1989; Riley, 1997).

In practice, however, many American children do not have access to the curriculum, teaching, and resources needed to help them reach high learning standards (Darling-Hammond, 2003; Darling-Hammond & Falk, 1997; Heubert, 2002). According to a study commissioned by the National Research Council, the failure to provide all students with a rich, high-quality mathematics curriculum and qualified teachers results in large proportions of students failing mathematics and thus abandoning its study early in their high school careers (Pendergast, 1989).

“Dropping out” of mathematics has short- as well as long-term consequences for students. Horn and Nunez (2000) reported that students whose parents did not attend college more than doubled their chances of enrolling in 4-year colleges if they took high school mathematics courses beyond Algebra 2, a second course in algebra. Similarly, a U.S. Department of Education study (Adelman, 1999) revealed that taking advanced mathematics in high school was more strongly associated with successful completion of college than any other factor, including high school grade point average and socioeconomic status (SES). Furthermore, a positive relationship has been shown between enrollment in advanced mathematics and higher earning power, even after factors such as occupation, demographic characteristics, and highest degree earned have been controlled (Rose & Betts, 2004). In other words, studying mathematics beyond Algebra 2 in high school strongly correlates with future educational and financial success (Adelman, 1999).

In a period in which the national aspiration is to ensure that all students meet high academic standards, it is incumbent upon schools to identify ways to help all students meet these standards. For some, the solution is low-track classes with a slower paced curriculum for low achievers and high-track classes with enriched and accelerated instruction for high achievers. Supporters of tracking believe that low-track placements help struggling learners and that grouping students heterogeneously diminishes the learning of high-achieving students (Loveless, 1998). Opponents believe that tracking itself, or at least low-track placement, is a root cause of persistent lower achievement (Heubert & Hauser, 1999; Kifer, 1993; McKnight, 1987; Oakes, Ormseth, Bell, & Camp, 1990) and that low-track placements are inimical to achieving high standards for all (Heubert, 2002). They point to studies of accelerated instruction indicating that an enriched, accelerated curriculum is more effective than a traditional remedial curriculum as a means of increasing the achievement of initial lower achievers (Bloom, Ham, Melton, & O’Brien, 2001; Levin, 1988; Oakes et al., 1990; Peterson, 1989; Singham, 2003; Slavin & Braddock, 1993). In other words, the belief that underlies their work is that a school’s accelerated and enriched “best curriculum,” reserved for its highest achievers, is the best curriculum for all students.

Background

Acceleration in Mathematics

In their initial form, accelerated classes represented an attempt to meet the needs of “gifted and talented” learners. The tradition of accelerated classes began in the 1890s with programs such as the Cambridge Double Track Plan of 1891 and the special progress classes of the New York City Schools (J. A. Kulik, 1992). In New York, the state board of regents mandated in 1984 that all school districts offer an accelerated curriculum in middle schools, including the study of Sequential Mathematics I (a course in algebra) in the eighth grade, to meet the needs of “gifted” students (New York State Education Department, 1984).

More recent research suggests that schools should provide a rigorous middle school mathematics curriculum to *all* students, not only initial high achievers. Analyses of international studies such as the Second International Mathematics Study (SIMS) and the Third International Mathematics and Science Study (TIMSS) led to surprising conclusions. Data from SIMS and TIMSS indicate that a traditional low-track, remedial curriculum actually depresses the mathematics performance of American students rather than improving it (Cogan, Schmidt, & Wiley, 2001; Kifer, 1993; Schmidt, 1993; Schmidt, Houang, & Cogan, 2002; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999). Such findings regarding American mathematics curricula led SIMS scholar Edward Kifer (1993) and TIMSS scholar William Schmidt (2004) to propose that all students take an algebra-based course in the eighth grade.

Other studies strongly support the use of accelerated curricula for disadvantaged learners as an alternative to traditional remediation (Finnan & Swanson, 2000; Levin, 1988, 1997). For example, in a study conducted in three demographically similar Utah school districts, John M. Peterson (1989) compared the effects of differentiated junior high mathematics programs among students identified as remedial (low achievers), average, or accelerated (high achievers). Seventh-grade low achievers were divided into three groups and placed into a remedial low-track class, a class with an average curriculum, or a heterogeneously grouped, accelerated pre-algebra course. The low-achieving students who were placed in the accelerated pre-algebra class showed significant improvement in mathematics skills as compared with those in the low-track program or the regular seventh-grade curriculum intended for average achievers.

Similarly, White, Gamoran, Porter, and Smithson (1996) evaluated the effects of replacing general mathematics track courses with college-preparatory mathematics courses in urban high schools serving large numbers of low achievers. While collecting data, the authors discovered a high degree of track “misplacement” in the Rochester, New York, high schools. Track misplacement resulted in some low achievers being placed in high-track mathematics classrooms. They found that if average achievers, which they referred to as C+ students, were misplaced into general mathematics, the course for the low achievers, their chance of completing the two college-preparatory

mathematics courses was 2%. If they were placed in the “stretch” class, designed for average achievers, their chance of completion rose to 23%. However, if they had been misplaced into the high-track course class reserved for high achievers, their chance of completing the 2-year regents sequence rose dramatically—to 91%. In other words, students had the greatest success if they received accelerated instruction in the high-track class.

Mason, Schroeter, Combs, and Washington (1992), working in an urban Missouri junior high school, studied what happened when students of average mathematics achievement were assigned to an advanced eighth-grade pre-algebra class. They found that the achievement of accelerated average students was better than the achievement of similar students in previous years who had not taken accelerated mathematics (Mason et al., 1992, p. 592). Moreover, the average-achieving students in the high-track classes enrolled in more advanced high school mathematics courses than did students at similar achievement levels from previous low-track cohorts. The results of the studies just described are consistent with the findings of Kifer, Wolfe, and Schmidt (1993), who demonstrated almost no growth among students placed in low-track, remedial eighth-grade classrooms, and the conclusions from a study commissioned by the National Research Council that documented strong negative effects of low-track classes (Heubert & Hauser, 1999).

Effects of Tracking on High-Achieving Students

But what happens to the achievement of initial *high* achievers when all students enter heterogeneous, high-track, accelerated mathematics classrooms? Some argue that elimination of tracking and an influx of low achievers will lead to reduced student achievement, especially in the case of the most talented students (Loveless, 1998, 1999). However, Mason et al. (1992) found that the performance of high-achieving students remained the same on the Comprehensive Assessment Program Achievement Series Test, a norm-referenced examination, after they had studied in heterogeneously grouped classes. In other words, high achievers did not learn less when they studied alongside their average-achieving peers than when they took mathematics in classes consisting entirely of high achievers. In fact, on one measure, concepts, the performance of high achievers in heterogeneous classes was significantly higher than that of previous cohorts educated in homogeneous classes (Mason et al., 1992, p. 593).

In addition, what are the effects when high achievers take mathematics not only with average achievers but with low achievers as well? White et al. (1996, p. 304) cautioned that the possible impact of a “sudden influx of low-achieving students” on the performance of traditionally high-achieving students is unknown. Indeed, the literature on tracking has produced mixed results regarding the performance of high-achieving students when they are grouped in heterogeneous classes with low-achieving students. Some studies have shown that high achievers are harmed by heterogeneous grouping (Brewer, Rees, & Argys, 1995; Epstein & MacIver, 1992; J. A. Kulik, 1992) and

that their performance is enhanced by tracking. Other studies have revealed no significant differences in the performance of high achievers when they are grouped in heterogeneous classes that include both average and low achievers (Figlio & Page, 2002; Mosteller, Light, & Sachs, 1996; Slavin, 1990).

These inconsistent results may be due to the fact that it is usually impossible to disentangle the effects of tracking itself from the effects of differentiated curricula and other factors associated with tracking (Kerckhoff, 1986; Lucas, 1999; Slavin & Braddock, 1993). In other words, if high achievers do learn less in heterogeneous classrooms, is it because average-achieving and low-achieving students are also present in these classrooms or because the curriculum in heterogeneous classes may be less demanding?

Those who support tracking argue that if the quality of both curriculum and instruction in low-track classes were improved and if more equitable student sorting practices were put in place, the negative effects of tracking on low-achieving students could be ameliorated (Gamoran & Weinstein, 1998; Hallinan, 1994; Loveless, 1998). According to this interpretation of tracking studies, the negative effects of tracking result from unfair allocation of resources to low-track classes, not from grouping practices. However, Heubert and Hauser (1999, p. 102), in a study commissioned by the National Research Council, reviewed the relevant published research and found no reported examples of typical public schools in which students received high-quality instruction in low-track classes. Other researchers believe that heterogeneous grouping works when schools ensure that all students are exposed to strong teaching and a demanding curriculum, and the studies just described provide empirical support for this view. Oakes (1982, 1986), for example, found that students in high-track classes receive higher quality instruction and that lessons in high-track classes include higher level thinking skills, with more time spent on instruction rather than drill-and-practice activities. Oakes believed that any higher achievement associated with high-track classes results not from grouping practices per se but from the advantages associated with other instructional factors associated with high-track classes, such as better instruction and a rigorous curriculum.

Whether all students can benefit equally from studying the same, best curriculum, such as the accelerated mathematics curriculum, is at the heart of the debate on tracking. The effects of differentiated curricula have not, thus far, been separated from the effects of tracking, and those on both sides of the debate argue that the interaction between tracking and curriculum supports their position.

Purpose of This Study

The goal of our study, then, was to fill some of these important gaps in the present literature. First, would more students take and pass such courses at the level of trigonometry and beyond if they took an accelerated algebra course in the eighth grade? Second, would the performance of initial higher achievers decrease if all students were heterogeneously grouped and accelerated in mathematics? We sought answers to these important questions.

We describe the results of a longitudinal study of the effects on student achievement when demographically diverse students in heterogeneous classes were given an accelerated high-track mathematics curriculum in Grades 6–8. Specifically, we examined how acceleration in heterogeneous middle school mathematics classes affected performance in advanced mathematics courses later selected and passed by students in high school. We also analyzed the effects of heterogeneous grouping in middle school mathematics on initial high-achieving students' performance on a New York State Board of Regents examination in mathematics and their subsequent performance on the advanced placement calculus exam. Thus, this study's conclusions inform the debate on the efficacy of tracking as an instructional strategy and add to the emerging literature on accelerated study as an alternative to remediation.

Context of the Study

The School District

The study site was a suburban community of 28,000 in Nassau County, Long Island, with a student population of approximately 3,500. Most students who attend the community's schools are White, and their families earn upper-middle-class incomes. Average enrollment in the high school is about 1,100 students. Approximately 8% of students are African American, 12% are Latino, and 2% are Asian. Most African American students are eligible for free or reduced-price lunches and live in Department of Housing and Urban Development apartments designated for low-income families. The majority of Latino families reside in Section 8 subsidized apartments in the downtown area. During the study period, approximately 145 high school students (13%) each year were eligible for free or reduced-price lunches; of these, more than 98% were students of color.

In the 1980s, the New York State Board of Regents replaced the 3-year regents course sequence of algebra, geometry, and trigonometry with a 3-year sequence of courses labeled Sequential Mathematics I, II, and III. Most of the Sequential Mathematics I curriculum was algebra. Sequential Mathematics II focused on topics in geometry, and Sequential Mathematics III focused on advanced algebra and trigonometry. In support of this curricular change, the state established examinations for each sequential course to measure student learning in mathematics.

When these new mathematics courses began, the state board of regents also required all districts to accelerate some students in mathematics and allow them to study Sequential Mathematics I before the ninth grade. The purpose was to allow students either to graduate early or to earn college mathematics credit while in high school (Maggio, 1988). Thus, here "accelerated mathematics" refers to a program of mathematics study that (a) teaches the usual sixth-, seventh-, and eighth-grade curricula in 2 years rather than 3 and (b) teaches the usual ninth-grade curriculum, an algebra-based course labeled Sequential Mathematics I, in the eighth grade.

Although the state board of regents mandated that districts provide acceleration, it allowed them to decide who should be accelerated. Originally, the study district allowed only the 50 most proficient mathematics students to accelerate in mathematics, and this decision was made in the fifth grade. In the late 1980s, as the district increased the number of students it accelerated, student performance remained high even with the more permissive policy. In the late 1990s, the district allowed any student to take accelerated mathematics, regardless of whether the student met the school's entrance criteria.

Acceleration was now theoretically available to all students, but in practice different ethnic populations were not choosing to accelerate at the same rates. For example, during the 1996–1997 school year, only 11% of African American students and 15% of Latino students were accelerated in eighth-grade mathematics, in comparison with an overall acceleration rate of 50% among White and Asian students. Given the great benefits of studying advanced mathematics, the district was concerned that many students, especially disproportionate numbers of students of color, were not taking part. Encouraged by the success of students who chose to accelerate, the administration concluded that it was in the best interest of all students to eliminate grouping for instruction and to provide mathematics acceleration for all.

The Change Process

The district developed a multiyear plan to eliminate tracking in mathematics at the middle school level (Grades 6–8). In addition, it instituted changes in teaching and learning conditions that school leaders believed would help all students succeed. These changes involved the following: (a) revision of the curriculum in Grades 6–8, (b) creation of alternate-day support classes known as mathematics workshops to assist struggling students, (c) establishment of common preparation periods for mathematics teachers, (d) integration of calculators, and (e) a revised mathematics teacher schedule consisting of four accelerated classes and two mathematics workshops.

The district decided that all tracking for instruction in the middle school would end with the sixth-grade class that would enter in 1995 and that all subsequent sixth graders would study accelerated mathematics in heterogeneously grouped classes. The superintendent and the middle school leadership team believed that the combination of (a) heterogeneous grouping, (b) a high-track curriculum, and (c) mathematics workshops would enable all learners to be successful without reducing the achievement of the most proficient students.

Students were placed in the alternate-day mathematics workshops according to teacher recommendations or parent requests. Workshop class sizes averaged eight students, and students were allowed to enroll in or leave the class on the basis of how they were doing in their regular class and their personal desire for support. All work in these classes supported instruction in the regular mathematics classroom, and, whenever possible, students were assigned

to a workshop taught by their regular mathematics teacher. Approximately 25% of all students took a workshop class at some time during the year, including a number of high-achieving students who wanted the additional instruction.

In fall 1997, the first cohort of accelerated students began the Sequential Mathematics I course in eighth grade. Throughout the year, the assistant principal and the superintendent met with teachers to discuss student progress and offer support. After the Sequential Mathematics I regents exam had been administered and graded, the district acknowledged universal acceleration as a success. Nearly all students (90%) in this first cohort of universally accelerated students passed the Sequential Mathematics I regents exam, and 52% of all test takers scored at the mastery level, with grades of 85% or above. The median score on the exam was 86%. This first cohort entered the district's high school in September of 1998 and graduated in June of 2002. Every cohort member met or exceeded the graduation standard in mathematics.

In the following, we describe our analyses and present the findings of our longitudinal study examining achievement data (in Grades 5–12) in six student cohorts: three cohorts from immediately before the reform discussed earlier and the first three cohorts after the reform. We include findings for all students as well as findings disaggregated according to ethnicity, poverty, and initial mathematics achievement.

Research Questions

As mentioned, two research questions guided this study. First, did more students take and pass more mathematics courses at the level of trigonometry (Sequential Mathematics III) and beyond after all students completed an accelerated algebra course in the eighth grade? Because the literature has described the effects of tracking on different groups of learners, this question was explored for all learners as well as the following subgroups: (a) initial low achievers, (b) initial average achievers, (c) initial high achievers, (d) African American or Latino students (who are typically underrepresented in accelerated mathematics classes) (Slavin & Braddock, 1993), and (e) low-SES students (students receiving free or reduced-price lunches).

The second research question focused on what has been an important and contentious aspect of the tracking debate: whether the achievement of initial high achievers is diminished by the inclusion of all learners in heterogeneously grouped mathematics classes. Initially, we examined the scores of initial high achievers on the Sequential Mathematics I regents examination. The scores of initial high achievers who took the regents examination after having taken the course in tracked (homogeneous) classes were compared with the scores of initial high achievers who took the course in heterogeneously grouped classes after universal acceleration.

In addition, to ascertain the intervention's long-term effects, we examined how well initial high achievers performed on the advanced placement calculus examination both before and after universal acceleration. Finally,

we examined the advanced mathematics course-taking patterns of initial high achievers before and after universal acceleration.

Method

Data and Design

To determine the effects of universal acceleration, we examined the mathematics achievement data of students in six consecutive annual cohorts. Our quasi-experimental design was modeled after interrupted time series designs (Cook & Campbell, 1979) in that the first three cohorts were not exposed to the intervention that the final three cohorts received.

The first three cohorts entered high school in 1995, 1996, and 1997, the 3 years before universal acceleration. These students were tracked in mathematics beginning in the sixth grade. The final three cohorts, which entered the high school in 1998, 1999, and 2000, were the first three in which all students were accelerated and heterogeneously grouped in Grades 6–8. During the years of this study, the district's racial demographics and socioeconomic cohort population characteristics remained stable, as did other factors; there were few changes in the mathematics teaching staff or teaching assignments, few changes in the administrative staff, and no changes in textbook series.

Selection effects are possible, however, even in stable populations. For example, the inclusion of transfer students whose educational histories differ from the majority could bias a study's results. A strategy for dealing with such effects is to include only data for the cohort members who have the most similar histories (Cook & Campbell, 1979). To reduce this possible source of bias, we included student data only for cohort members (entering high school in 1995–2000) in regular education who (a) were continuously enrolled in the school district from fifth grade to their exit from high school or completion of this study, (b) entered ninth grade between 1995 and 2000, and (c) had a permanent record folder containing all data of interest.

Data of interest included the following: (a) fifth-grade stanine score on the Iowa Test of Basic Skills (ITBS) Mathematics Concepts subtest (NSTANINE); (b) initial score on the Sequential Mathematics I regents examination (INIT); (c) if taken, score obtained on the advanced placement calculus examination (CALCSCOR); (d) mathematics courses taken in Grades 8–12 (H8–H12); (e) year of entry into the ninth grade (PREPOST); (f) ethnicity (i.e., African American or Latino) (UNDEREP); and (g) free or reduced-price lunch status (LOWSES).

Students who were not continuously enrolled in Grades 6–12 were excluded to ensure that students in the study had similar mathematical histories. The process of mathematics acceleration begins in the sixth grade in this district. With the exception of the variation introduced by having different teachers, all cohort members in mathematics classes of continuously enrolled students in the same track received similar instruction in a common, school-developed curriculum. In addition, the entrance criteria for advanced mathematics courses were the same for continuously enrolled students.

There was a compelling reason to exclude special education students as well. All members of the final two cohorts, with the exception of developmentally delayed students, were prepared to take the Sequential Mathematics I regents examination. In the first four cohorts, however, not all special education students were prepared to take the exam. This inconsistency had the potential to bias the results, making it appear as though the treatment (universal acceleration) had a more profound effect than it actually had on the taking of advanced mathematics courses; therefore, only regular education students were included. After application of these criteria, each of the six cohorts ranged between 152 and 181 students. There were 477 pre-universal-acceleration participants and 508 post-universal-acceleration participants.

Measures of Achievement

We used four measures of student achievement: fifth-grade stanine scores on the ITBS Mathematics Concepts subtest (NSTANINE), scores obtained by students the first time they took the Sequential Mathematics I regents exam (INIT), (c) students' scores on advanced placement calculus exams (CALCSCOR), and mathematics courses taken and passed by students in high school (H8–H12). Stanine scores were used to control for initial achievement; the other three measures were used as achievement outcomes.

Iowa Test of Basic Skills

The ITBS is a battery of tests created to measure the extent to which students in grades K–8 have acquired basic skills. In the case of all six cohorts in the study, the school district used Form J of the primary basic battery to measure the initial achievement of fifth graders. In mathematics, the subtests were Concepts, Problem Solving, and Computation. We used national stanine scores on the Concepts subtest as an indicator of students' fifth-grade achievement levels. Maggio (1988) identified scores on this subtest as a predictor of performance on the Sequential Mathematics I regents examination. These stanine scores were also used to form three groups of students for this study: (a) initial low achievers (Stanines 2–4), (b) initial average achievers (Stanines 5–7), and initial higher achievers (Stanines 8 and 9).

ITBS stanine scores were the only nationally standardized scores available in students' record folders and, other than teacher grades, the only available measure of student achievement after Grade 2 and before the regents examination. In some cases, not all of the study elementary schools retained information on Mathematics Concepts subtest scores for all six of the study cohorts.

The stanine scale has values that range from 1 to 9, with a mean value of 5 and a standard deviation of 2. The stanines used are nationally normed and thus comparable across both the fall and spring sittings of the ITBS test. It should be noted that an ordinal measure such as a stanine does not allow for the finer control of initial achievement that more continuous measures

would supply, nor does it allow for the identification of students at both tails of the distribution—for example, the top 5% and the bottom 5% in terms of initial achievement.

Sequential Mathematics I Regents Examination

The Sequential Mathematics I regents examination, prepared by a committee of New York State teachers in conjunction with education department specialists in mathematics and testing (New York State Education Department, 2001), was administered until January of 2002. The purpose of the exam was to measure student knowledge of the Sequential Mathematics I curriculum, an algebra-based curriculum that also included introductory topics in geometry and statistics. Students took the test during a single 3-hour administration each January, June, and August, at a time designated by the New York State Board of Regents.

Scores on this examination were expressed as percentages of items correct based on point values. Scores ranged from 0%–100%. Scores of 65% and above were designated as passing by the New York State Board of Regents, and scores of 85% and above were designated as reflecting topic mastery. If students do not pass a regents examination, or if they are not satisfied with their score, they can retake the exam.

Advanced Placement Calculus Examination

Advanced placement examinations in both AB and BC calculus are provided by the College Board and prepared and graded by the Educational Testing Service. Advanced placement examinations are administered in May of each year at a time determined by the College Board. Scores range from 1 to 5. Students in the study school self-selected between the two courses. Because both the BC and AB exams produce an AB score, the AB score was used as the outcome measure of achievement. All students who were enrolled in the course took the advanced placement exam.

High School Mathematics Courses

Successful completion of a mathematics course was operationally defined as passing the course with a minimum final grade of 65% and, in the case of Sequential Mathematics I, II, and III, passing the externally developed New York State regents examination for the course with a grade of 65% or better. The secondary mathematics course sequence was as follows: (a) Sequential Mathematics I, (b) Sequential Mathematics II, (c) Sequential Mathematics III, (d) courses in precalculus, and (e) advanced placement calculus (AB or BC). Completion of Sequential Mathematics III, a course in advanced algebra and trigonometry, has been shown to be associated with later college success (Adelman, 1999).

There were two tracks beginning in the ninth grade for all of the study cohorts, including the universally accelerated cohorts. Students selected

between honors courses and regents courses. Some mathematics courses are designated “H” (honors) at the high school; others are designated “regents” courses. Honors courses include more mathematics topics and are more demanding than those designated as “regents.” In the case of the last three cohorts, all courses preceding Sequential Mathematics II were heterogeneously grouped, with one level of study only.

Analysis of First Research Question

We conducted two analyses to determine whether more students in the district of study took and passed increased numbers of mathematics courses at the level of trigonometry and beyond after universal acceleration. We first examined whether the proportion of students who successfully completed high school mathematics courses one grade level earlier than the average New York State student (the non-accelerated student) had increased after detracking. Data for the three pre-universal-acceleration cohorts were combined, as were data for the three post-universal-acceleration cohorts.

Proportions were computed for all students as well as for the following subgroups: (a) initial low achievers (students with fifth-grade mathematics stanine scores of 2–4), (b) initial average achievers (stanine scores of 5–7), (c) initial high achievers (stanine scores of 8–9), (d) African American or Latino students, and (e) low-SES students. There were 62 students in the initial low achievement group, 525 students in the initial average achievement group, and 398 students in the of initial high achievement group.

Because special education and transient students did not meet the criteria for inclusion described earlier, the number of initial low achievers included in the study was low. Because Stanines 8 and 9 include scores at the 80th national percentile and above, they were used as the criterion for identifying initial high achievers. In other studies, reported rates of eighth-grade student acceleration and enrollment in algebra have varied from 20% to 25% (Epstein & MacIver, 1992; Goycochea, 2000; Horn & Nunez, 2000).

Although descriptive statistics were useful as a first level of analysis, it was important to ensure that other factors, such as increased initial achievement of the cohorts or changes in the demographics of the district, were not responsible for any changes in regard to increased numbers of students taking and passing advanced mathematics classes. Therefore, in our second level of analysis, we used binary logistic regression to compute the probability of students taking (a) Sequential Mathematics III, (b) a precalculus course, or (c) an advanced placement calculus course. Logistic regression analysis allowed us to control for key variables often associated with school achievement—specifically, previous mathematics achievement, SES, and ethnicity.

To control for previous achievement, we used stanine scores on the fifth-grade ITBS Mathematics Concepts subtest (NSTANINE). The three other independent variables were PREPOST, UNDEREP, and LOWSES. We analyzed each of the three advanced mathematics levels separately.

Analysis of Second Research Question

The second research question addressed the effects of universal acceleration in heterogeneously grouped classes on the performance of initial high-achieving students. National stanine scores of 8 or 9 on the fifth-grade ITBS Mathematics Concepts subtest were used to identify students as initial high achievers.

We first compared the scores of pre-universal-acceleration cohorts of initial high achievers, who studied mathematics in tracked classes, with those of post-universal-acceleration cohorts of initial high achievers who studied mathematics in detracked, accelerated classes in Grades 6–12. The measure of achievement was their performance on the Sequential Mathematics I regents exam (INIT). We used a two-tailed *t* test of means to determine whether or not the two populations (initial high achievers in the pre- and post-universal-acceleration cohorts) were significantly different.

Students from the first three cohorts were paired with students from the second three cohorts according to (a) national stanine scores on the ITBS Mathematics Concepts subtest, (b) racial/ethnic group, and (c) SES. Because there were more initial high achievers in the final three cohorts, we created matched pairs using randomly assigned identification numbers sorted in descending order. Pairings were made until all were exhausted. Degrees of freedom were equal to the number of pairs minus 1.

We then compared the advanced placement calculus scores (CALC-SCOR) of the post-universal-acceleration cohort members with the scores of the pre-universal-acceleration cohort members using regression analysis. The goal was to determine whether there were any longitudinal effects on high achievers due to universal acceleration and the increase in the number of students at lower initial achievement levels in calculus classes. The same fifth-grade stanine scores were used to control for initial achievement.

Results

Increases in Advanced Mathematics Study

Examination of our first research question showed that the percentage of students taking advanced math courses did increase after universal acceleration. More students took advanced mathematics classes, more students passed such courses along with their associated New York State examinations, and more students completed such courses a year sooner than the average student in New York State.

Table 1 shows the percentages of students in all six cohorts, overall and by initial achievement level, studying and passing each level of mathematics at each grade level. By the end of 12th grade, 92% of all students in the post-universal-acceleration group had passed a course and the regents examination in Sequential Mathematics III. In addition, 85% had passed a course with a precalculus curriculum. At the same time, the percentages of students who did not take mathematics or who took mathematics courses below their grade levels *decreased* after the reform (9th grade: 8% before, 2% after;

Table 1
**Percentages of Students Successfully Completing
 Mathematics Courses, by Grade**

Course	Overall		Low achievers (<i>n</i> = 62)		Average achievers (<i>n</i> = 525)		High achievers (<i>n</i> = 398)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Grade 8								
Seq I	51	96	5	60	39	95	78	100
Grade 9								
Seq I	92	98	57	76	93	98	99	100
Seq II	50	89	5	44	38	85	77	99
Grade 10								
Seq I	95	98	76	84	95	98	99	100
Seq II	87	94	46	64	85	93	98	99
Seq III	49	83	5	28	38	77	76	97
Grade 11								
Seq I	96	99	81	92	96	99	100	100
Seq II	90	97	57	80	90	97	98	100
Seq III	80	91	30	40	78	88	94	99
Precalculus	48	81	5	24	37	75	75	94
Grade 12 ^a								
Seq I	96	99	81	94	96	99	100	100
Seq II	92	98	65	88	92	97	99	100
Seq III	83	92	38	53	81	91	89	99
Precalculus	70	85	19	35	66	84	81	96
Advanced placement calculus	40	56	5	18	29	44	64	76

Note. Pre = pre-universal acceleration; Post = post-universal acceleration; Seq = sequential mathematics. *N* = 985.

^aPercentages for post-universal-acceleration cohorts exclude the members of the final cohort, who were in Grade 11 at the completion of the study.

10th grade: 13% before, 6% after; 11th grade: 20% before, 9% after; 12th grade: 30% before, 15% after). Thus, the increased rigor did not lead students to stop taking mathematics courses beyond the school requirement, as some had feared.

Effects on Subgroups of Learners

Studies of tracking and acceleration indicate that changes in instructional grouping affect learners differently depending on factors such as previous

achievement levels, often characterized in the literature as “ability.” To determine whether overall results masked differing effects on different groups of learners, we used stanine scores on the Mathematics Concepts subtest to group students into one of the three subgroups described earlier: initial low achievers, initial average achievers, or initial high achievers. As can be seen in Table 1, after the district detracked and accelerated all middle school students, more students at all three initial achievement levels completed advanced mathematics courses before graduating. Moreover, the percentages of students at each initial achievement level no longer taking mathematics, or taking mathematics courses below grade level, were lower.

We also examined the short- and long-term effects of an accelerated curriculum on African American and Latino students (UNDEREP) and on students who received free or reduced-price lunches (LOWSES). Before the reform, middle school mathematics classes were stratified according to race and SES because underrepresented groups studied the accelerated curriculum in disproportionately low numbers. Such patterns are common throughout the United States.

Part of this discrepancy can be attributed to the lower average initial achievement of African American, Latino, and low-SES students in the district of study. Significantly, however, there were gaps by race and SES even when students had the same high initial achievement levels. For example, among students whose ITBS Mathematics Concepts stanines were 6 or greater, 63% of White and Asian students—but only 35% of African American and Latino students—took accelerated mathematics in eighth grade. In short, although all students in the pre-acceleration cohorts had the option of taking accelerated mathematics, African American students and Latino students took advantage of this option at a much lower rate than White or Asian students at similar achievement levels. The effects of universal acceleration on these two groups, in terms of mathematics courses completed, are presented in Table 2.

Some findings are of note regarding minority and low-SES students. For example, after universal acceleration in heterogeneously grouped classes, the percentage of minority students who met the mathematics commencement requirement (passing the Sequential Mathematics I regents examination) before they entered high school tripled, from 23% to 75%. Also, higher percentages of African American, Latino, and low-SES students passed the exam in eighth-grade detracked classes than in tracked eighth- and ninth-grade classes before universal acceleration. Moreover, two thirds of African American, Latino, and low-SES students in the post-universal-acceleration cohorts successfully completed Sequential Mathematics III, the first advanced mathematics course identified in the literature as being associated with success in college (Adelman, 1999).

Finally, it is important to note that although a gap remains between White/Asian students and African American/Latino students in terms of study of advanced mathematics, it has narrowed. Before universal acceleration, 88% of White and Asian students had passed the Sequential Mathematics III course

Table 2
**Percentages of Students Successfully Completing
 Mathematics Courses, by Grade**

Course	African American/Latino students (<i>n</i> = 121)		Low-SES students (<i>n</i> = 61)	
	Pre	Post	Pre	Post
	Grade 8			
Seq I	23	75	13	69
	Grade 9			
Seq I	72	84	45	76
Seq II	22	59	13	55
	Grade 10			
Seq I	83	87	68	83
Seq II	52	67	35	66
Seq III	18	41	13	45
	Grade 11			
Seq I	88	95	77	97
Seq II	58	84	35	86
Seq III	42	56	32	48
Precalculus	18	39	13	34
	Grade 12 ^a			
Seq I	84	95	77	90
Seq II	67	86	52	86
Seq III	46	67	32	67
Precalculus	33	52	32	43
Advanced placement calculus	11	36	10	29

Note. Pre = pre-universal acceleration; Post = Post-universal acceleration; Seq = sequential mathematics. *N* = 182.

^aPercentages for post-universal-acceleration cohorts exclude the members of the final cohort, who were in Grade 11 at the completion of the study.

and the regents exam, while only 46% of African American and Latino students had done so. After universal acceleration, the respective percentages were 95% and 67%—both groups improved, and the gap narrowed considerably.

Binary Logistic Regression Analysis

Although the descriptive statistics indicate the positive effect of universal acceleration on all learners, as mentioned, it was important to ensure that

other factors such as differences in initial achievement and changes in school demographics were not responsible for the increased numbers of students taking and passing advanced mathematics classes. Binary logistic regression analysis allowed us to include other factors known to affect achievement in a model estimating the probability that an outcome would occur. In this case, the outcome was the probability of students taking specific mathematics courses.

In calculating the probability of students taking Sequential Mathematics III, precalculus, and advanced placement calculus, we included covariates often associated with school achievement—specifically, previous mathematics achievement, SES, and ethnicity—as well as the PREPOST variable, which identified students who were members of universally accelerated cohorts. The likelihood ratio statistic and the Wald statistic produce p values at various levels of significance (Menard, 1995). We analyzed each of the three advanced mathematics levels separately.

Overall Fit of the Model

In comparing the $-2 \log$ -likelihood statistics of a model with and without the variables of interest, binary logistic regression analysis determines whether the model that includes the selected covariates is a good fit, that is, whether the model that includes the independent variables, or covariates, is a better predictor of the outcome than is a model that does not include these variables (Hosmer & Lemeshow, 2000). For all three advanced levels of mathematics courses, the model that included initial achievement (NSTANINE), membership in an accelerated cohort (PREPOST), membership in a previously under-represented group (specifically, African American or Latino) (UNDEREP), and LOWSES was better than the model that included only the generated constant. With these variables as predictors, the chi-square statistic of significance predicting the probability of completing advanced mathematics courses was significantly different from zero at the .0001 level for each advanced level of mathematics.

In the case of Sequential Mathematics III, all four covariates contributed significantly to the model. In the case of precalculus and advanced placement calculus courses, low SES was not a significant contributor. However, when UNDEREP (1 = African American or Latino) was removed from the model, LOWSES became a significant predictor. Because there was a large statistical overlap in the district between African American and Latino students and students who received free or reduced-price lunches, the lack of significance of LOWSES in the model with UNDEREP is not surprising, nor does it necessarily indicate that there is no relationship between poverty and enrollment in high-level mathematics courses. The simplest explanation is that, in this district, race and poverty are collinear and overlap to such a degree that one accounted for the other in the model (Menard, 1995).

The finding of importance in regard to the first research question was the following: For all three levels of advanced mathematics courses, membership

in a post-universal-acceleration cohort (PREPOST = 1) was a significant contributor to the probability of a student taking an advanced mathematics course, even when initial achievement, low SES, and ethnicity were controlled. In short, after all students had been detracked and studied a regents algebra-based course in the eighth grade, the probability of a student completing an advanced mathematics course before graduating from high school significantly increased. In the following section, we highlight the contributions that universal acceleration made to students' subsequent course taking in mathematics.

Contribution of Covariates

Logistic regression analysis also provides rich information regarding the contributions of individual covariates to the probability of a studied outcome. As does the linear regression coefficient, the logistic regression coefficient indicates the strength of each variable's contribution to the dependent variable, which, in this case, was the probability of taking an advanced mathematics course. Along with logistic regression coefficients (B values), standard errors, R values, and log odds ratios (shown as $\text{Exp}[\beta]$), Wald chi-square statistics (which measure the contribution of each covariate when the remaining covariates are held constant) are presented in Table 3.

Both initial achievement and underrepresented status significantly contributed to predicting whether or not students would complete advanced mathematics courses. However, when these two factors were held constant, universal acceleration also significantly increased the odds of a student taking and passing advanced mathematics. Calculation of the log odds ratio for each variable allowed us to ascertain the impact of accelerating all students in terms of the predicted odds of students taking and passing advanced mathematics courses.

An examination of Table 3 reveals that, according to model predictions, members of the universally accelerated cohorts were more than 2.6 times as likely as members of the pre-accelerated cohorts to take and pass Sequential Mathematics III, more than 2.7 times as likely to pass a course in precalculus, and 2.0 times as likely to take and pass advanced placement calculus. To put this increase in perspective, each step up the initial achievement ladder (as described via stanine scores) increases the odds of a student taking and passing an advanced mathematics course. The log odds ratio describes the increase. A comparison of the log odds ratios for NSTANINE with those for PREPOST, at all advanced mathematics levels, showed that the "boost" in regard to completing advanced mathematics courses provided by membership in an accelerated cohort exceeded the "boost" provided by a one-stanine-higher ITBS score.

Specific Probabilities

In addition to computing the individual contributions of covariates to odds of completing advanced mathematics courses, the model provided the logistic

Table 3
Binary Logistic Regression Analysis Results

Covariate	Sequential Mathematics III				Precalculus				Advanced placement calculus						
	Wald statistic	B	SE	R	Exp(β)	Wald statistic	B	SE	R	Exp(β)	Wald statistic	B	SE	R	Exp(β)
PREPOST	15.12***	0.97***	0.25	.13	2.63	29.27***	1.01***	0.19	.16	2.74	18.13***	0.69***	0.16	.12	1.99
NSTANINE	84.10***	0.76***	0.08	.33	2.15	97.68***	0.61***	0.06	.30	1.85	113.61***	0.61***	0.06	.31	1.85
UNDEREP	17.97***	-1.38***	0.33	-.15	0.25	47.51***	-1.62***	0.23	-.21	0.20	10.05**	-0.88**	0.28	-.08	0.41
SES	6.15*	-1.02*	0.41	-.07	0.36										
Constant	29.69	-2.77***	0.51			50.18***	-2.92***	0.41			116.88	4.62***	0.43		

Note. SES was not a significant contributor to the precalculus and advanced placement calculus models, and thus it was removed. -2 log-likelihood values for the three models were 502.31 (Sequential Mathematics III), 789.23 (precalculus), and 926.96 (advanced placement calculus). The corresponding chi-square values were 239.44 (Sequential Mathematics III), 252.05 (precalculus), and 191.49 (advanced placement calculus) (all p s < .0001). * p < .05. ** p < .01. *** p < .001. **** p < .0001.

regression coefficient for each covariate. We were able to use this coefficient to compute specific probabilities for individual cases, and as such we could estimate probabilities of completing advanced mathematics courses among students with specific demographic and initial achievement characteristics (Menard, 1995). Table 4 presents the resulting probabilities for possible combinations of stanine scores and demographic characteristics that were statistically significant contributors to the models. As an example, we were able to estimate that a low-SES African American student with a fifth-grade mathematics stanine of 7 had a 20% probability of completing Sequential Mathematics III before universal acceleration. After universal acceleration, that probability increased to 40%.

It can be seen from Table 4 that, for each initial achievement level measured according to stanine scores of 2–9 and for all groups of students, taking accelerated middle school mathematics in detracked classes resulted in an increased probability of taking and passing more advanced mathematics courses. Universal acceleration even increased the probability that students at higher initial achievement levels would complete advanced mathematics courses. To put these increases in the context of initial achievement, a close inspection of Table 4 shows that for all advanced levels of mathematics, membership in an accelerated cohort contributed more to the likelihood of completing an advanced mathematics course than did an initial achievement level a full stanine higher. This increase was greatest for precalculus.

Comparison With New York State

The interrupted time-series design allowed us to evaluate the statistical significance and magnitude of changes in course taking and achievement between the pre-acceleration and post-acceleration student cohorts. However, it did not ensure that the results obtained would not have been observed in the absence of acceleration. For example, perhaps the regents examination for the first advanced mathematics course (i.e., the Sequential Mathematics III regents exam) had become progressively easier over time, or more students had been induced to study advanced mathematics in New York State in response to the increasing pressures for higher standards. In other words, were the increases that we observed an outcome of a general trend in terms of increased advanced mathematics study, and thus would have probably occurred in the absence of the intervention? To assess this possibility, we compared changes over the pre-acceleration and post-acceleration periods with data for comparable schools.

Each year the New York State Education Department publishes data on the numbers of New York public high school students who take and pass regents examinations. In addition, it uses a formula to identify similar schools, that is, schools similar in regard to student body, geographic area, and resources (New York State Education Department, 2004).

For both the 1998–1999 and 2001–2002 school years, we compared data from the study school with data from six similar schools in the county. These

Table 4
Comparison of Probabilities by Ethnicity and Initial Achievement

Ethnicity	Fifth-grade mathematics stanine score																	
	2		3		4		5		6		7		8		9			
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post		
	Sequential Mathematics III																	
Asian/White	.06	.14	.12	.26	.22	.43	.38	.62	.57	.78	.74	.88	.86	.94	.93	.97		
African American/Latino	.02	.04	.03	.08	.07	.16	.13	.29	.25	.47	.42	.65	.61	.80	.77	.90		
African American/Latino: low SES	.01	.01	.01	.03	.03	.06	.05	.13	.11	.24	.20	.40	.35	.59	.54	.76		
	Precalculus																	
Asian/White	.05	.13	.09	.21	.16	.34	.25	.48	.39	.63	.54	.76	.68	.85	.80	.92		
African American/Latino	.01	.03	.02	.05	.04	.09	.06	.16	.11	.25	.19	.39	.30	.54	.44	.68		
	Advanced Placement Calculus																	
Asian/White	.01	.02	.02	.04	.03	.06	.06	.11	.10	.19	.18	.30	.28	.44	.42	.59		
African American//Latino	.00	.01	.01	.01	.01	.03	.03	.05	.05	.09	.08	.15	.14	.25	.23	.38		

Note. Pre = pre-universal acceleration; Post = post-universal acceleration.

2 school years were selected to ensure that there was no overlap between the pre-acceleration and post-acceleration groups. During the 1998–1999 school year, the students in the study high school who were eligible to take the Sequential Mathematics III regents exam (Grade 10 or beyond) were members of one of the three tracked cohorts before all students were accelerated in mathematics. During the 2001–2002 school year, all of the students were members of one of the three universally accelerated cohorts included in the study.

A key statistic of importance is the percentage of students taking and passing regents examinations according to average grade enrollment (AGE). AGE is computed by dividing the total enrollment of a school by the number of grades in the school. For instance, a 4-year high school with a total enrollment of 400 students would have an AGE of 100. This is critical information because it allows the public to estimate approximately how many New York State public high school graduates pass regents examinations. As an example, a school may have a regents examination passing rate of 100%, but if only 20% of the class takes the exam, the percentage of students passing the exam based on AGE would be only 20%. To compute a passing rate based on AGE, we combined data from the six comparison schools by (a) summing the number of students from each school who had passed the test and (b) dividing that number by the sum of each school's AGE.

An inspection of Table 5 reveals that, before universal acceleration, the percentage of Sequential Mathematics III test takers per AGE was higher in the study school than in the six comparison schools (79% vs. 68%). After all study school students had been accelerated, the increase in AGE-based exam enrollment was dramatically higher (105%) than the combined rate for similar schools (65%). At the same time, the AGE-based passing rate for the school dramatically increased (from 69% to 96%), whereas the AGE-based passing rate for the comparison schools declined (from 57% to 54%).

Table 5
**Comparison of Sequential Mathematics III Regents Examination
 Results: Study School and Six Similar Schools**

Measure	Similar schools		Study school	
	1998–1999	2001–2002	1998–1999	2001–2002
AGE	1,326	1,337	257	264
% AGE tested	68	65	79	105
% AGE passing	57	54	69	96

Note. The AGE percentage for the study school in 2001–2002 exceeded 100% for two reasons: (a) The size of the accelerated 10th-grade class ($n = 272$) exceeded the AGE ($n = 264$), and (b) students who had failed the exam during the previous year retook it during the 2001–2002 year.

AGE = average grade enrollment.

From these data, we conclude that the increase in the number of students in the accelerated cohorts taking and passing the Sequential Mathematics III regents examination was not part of a general regional trend in terms of increased advanced mathematics study. This finding strengthens our interpretation of our logistic regression analyses—that being a member of a universally accelerated cohort dramatically increased one’s probability of taking and passing an advanced mathematics course.

Effects of Universal Acceleration on Initial High Achievers

The second research question addressed the effects of universal acceleration taught in heterogeneously grouped classes on the performance of initial high-achieving students. National stanine scores of 8 or 9 on the fifth-grade ITBS Mathematics Concepts subtest were used as the criterion in identifying students as initial high achievers.

Performance on the Sequential Mathematics I Regents Examination

The first measure of posttreatment achievement consisted of scores on the Sequential Mathematics I regents examination (INIT). A two-tailed *t* test of means was used to determine whether the two populations (initial high achievers in the pre-universal-acceleration and post-universal-acceleration cohorts) were significantly different.

We found that the difference in mean scores was not significantly different from zero at the .05 level: 93.07 (pre-acceleration) versus 91.72 (post-acceleration). Because we used stanine scores to identify high achievers, we were not able to control for fine within-group differences. Therefore, we were not able to determine whether heterogeneous grouping affected the scores of particular subgroups (e.g., the top 2%). However, we can conclude that the scores of students in the district at the stanine level of 8 or 9 were not affected by heterogeneous grouping.

Performance on the Advanced Placement Calculus Examination

The second measure of posttreatment achievement consisted of scores on advanced placement exams in calculus (AB and BC). In this case, we found that universal acceleration was associated with increased achievement among (a) all students who took the exam and (b) high achievers who took the exam. As can be seen in Table 6, the scores of members of the post-universal-acceleration cohorts were significantly higher ($p < .01$) than those of members of the pre-universal-acceleration cohorts. Because the regression coefficient was .32 and the standard deviation was .99, the effect size associated with universal acceleration was an increase of one third of a point on a 5-point scale.

While stanine scores were significant predictors in the model that included all students, they were not significant predictors in the model including high achievers only. The effects just described were confirmed by a subsequent

Table 6
**Regression Analysis Results:
 Scores on Advanced Placement Examination**

Variable	All students (<i>N</i> = 283)			Initial high achievers (<i>n</i> = 156)		
	<i>B</i>	<i>SE</i>	β	<i>B</i>	<i>SE</i>	β
PREPOST	.32	.12	1.51**	.54	.19	0.22**
NSTANINE	.19	.04	2.51****	.17	.18	0.07
Constant	.28	.35		.37	.16	

Note. Model values were as follows: $R = .285$, $R^2 = .08$, $SE = .99$ for all students and $R = .228$, $R^2 = .05$, $SE = 1.15$ for initial high achievers.

** $p < .01$. **** $p < .0001$.

analysis in which 10th-grade Practice Scholastic Assessment Test scores were used to control for initial achievement. Again, advanced placement calculus scores were significantly higher in the post-universal-acceleration cohorts than in the pre-universal-acceleration cohorts ($p < .0001$). Among initial higher achievers—students with stanines of 8 and 9—the effect size increase was one half of a point.

In summary, we found no evidence that initial high achievers, defined as students whose stanine scores were in the top 20% nationwide, learned less when all students were accelerated in mathematics and had studied in untracked, middle school mathematics classes. On two important externally developed, secure examinations, scores of initial high achievers either improved or remained statistically unchanged after universal acceleration.

Additional Observations

Although students with fifth-grade stanine scores of 8 and 9 represented the largest proportions taking accelerated mathematics before the reform, we found that some students had “slipped through the cracks”; that is, even though they were capable of studying the accelerated high-track course (Sequential Mathematics D) in the eighth grade, they were counseled—or chose—not to take it. In fact, as shown in Table 1, before the reform only 78% of initial high achievers took and passed the accelerated, high-track course in eighth grade. Yet, after universal acceleration, all students with initial stanine scores of 8 or 9 on the fifth-grade subtest passed the accelerated course and regents exam before entering high school.

Although all groups of initial high achievers took more advanced mathematics courses after universal acceleration, the beneficial effects were greatest among students of color. Overrepresented among the initial high achievers who did not accelerate in the earlier cohorts, 7 of the 16 initial high-achieving African American or Latino students did not take the high-track mathematics

course. After universal acceleration, all high-achieving students of color took and passed both the course and the regents examination in the eighth grade. Even though the number of students was small, the present data provide additional evidence of the pattern of advantage that universal acceleration brought to the district's minority students. To summarize, after universal acceleration, (a) initial high achievers took more advanced mathematics courses, as did other groups; (b) more initial high achievers took advanced placement calculus exams, as did other groups; and (c) more initial high achievers earned higher scores on the advanced placement calculus exams, as did other groups.

Discussion

In their analysis of American students' achievement in the Third International Mathematics and Science Study, William Schmidt and his colleagues (1999) noticed how tracking creates differences in students' opportunities to study mathematics and how reduced opportunities result in lower achievement. Commenting on tracking in mathematics and science, they noted that although there will be differing access to courses as a consequence of state and local control of schools, there are other differences in access that stem from policies, such as tracking, that deny students access to educational opportunities. We examined how students' mathematics course-taking patterns and mathematics achievement were affected when the policy of tracking was abandoned and all students took accelerated mathematics study in heterogeneously grouped classes. In the following, we summarize findings of importance along with implications for practice, policy, and future research.

Universal Acceleration and the Study of Advanced Mathematics Courses

Atanda (1999), Horn and Nunez (2000), and Stevenson, Schiller, and Schneider (1994) found a strong relationship between the study of algebra in the eighth grade and the study of trigonometry and precalculus in high school. Our study, which assessed the long-term effects of all students studying algebra in the eighth grade, confirms these earlier results. We found that, after universal acceleration, more students continued to study those rigorous mathematics courses associated in the literature with later success in terms of both college completion and future earnings (Adelman, 1999; Rose & Betts, 2004). More students at all initial achievement levels studied advanced mathematics courses, including precalculus and advanced placement calculus. The percentage of post-universal-acceleration initial average achievers taking and passing a precalculus course exceeded the percentage of pre-universal-acceleration students who passed Sequential Mathematics III.

More students at initial lower achievement levels studied advanced mathematics as well. Increases were 38% to 53% for Sequential Mathematics III and 19% to 35% for precalculus. To put these increases in a national perspective, data from the National Center for Education Statistics (2002) show that only 26.7% of all high school graduates studied precalculus in 2000. After

universal acceleration, the study district's initial lowest achievers exceeded the national average.

Whereas previous studies of detracking have generally shown benefits for initial low and initial average achievers, we found that more high achievers are also taking advanced mathematics courses. Since the implementation of universal acceleration in heterogeneously grouped middle school mathematics classes, 99% of all initial high achievers in the post-acceleration cohorts have passed a trigonometry course and regents examination, and 96% have passed a precalculus course. Before universal acceleration, the corresponding percentages were 89% and 81%. It is noteworthy that *every* African American or Latino high-achieving student in one of the post-universal-acceleration cohorts has successfully completed a trigonometry course, has passed a New York State regents exam that measures skills in trigonometry and advanced algebra, and has gone on to study precalculus. This is in contrast with the pre-universal-acceleration-of-all years, when only 69% of high-achieving students of color completed precalculus.

Although the number of initial high-achieving minority students in this study was relatively low ($n = 23$), the increase in the proportion of such students successfully studying the school's most rigorous mathematics courses is indicative of the positive effects of detracking. Considering cases in which all students are exposed to the high-track curriculum, the present findings challenge the assertion that tracking may be advantageous to high-achieving African American students (Loveless, 1998, 1999).

To better understand the impact of the reform, it is helpful to place the achievement of minority students in the district within a national context. According to the National Center for Education Statistics (2002), the ethnic student group exhibiting the highest level of participation in advanced mathematics study is Asian/Pacific Islanders. Nationally, 48% of Asian/Pacific Islander students completed precalculus courses in 2000, and 24% completed advanced placement calculus courses. In the present study, African American and Latino students who were members of universally accelerated cohorts *exceeded* the national rates for Asian/Pacific Islanders: 52% studied and passed a precalculus course, and 36% studied advanced placement calculus.

The percentage of low-SES students studying and passing a trigonometry course and its regents examination more than doubled after the studied reform—from 32% to 67%. Although it is important to caution that our findings are based on a relatively small population of low-SES students ($n = 61$), such a large increase is promising. The percentage of low-SES students studying mathematics at the level of trigonometry or above in the universally accelerated cohorts (67%) exceeded the national percentage found by Atanda (1999) among graduating seniors of *high* SES (55.7%).

Finally, we found no evidence that increased numbers of students fell behind grade level or dropped out of mathematics as a result of this reform. On the contrary, such numbers declined. On the basis of examinations of raw proportions of students as well as logistic regression analyses that controlled for achievement and demographic characteristics, all student groups

appear to have benefited in important ways. Belying the claims of tracking proponents, there was no evidence that any group was worse off than before.

Effects of Heterogeneous Classes on the Scores of Initial High Achievers

A meta-analysis of experimental and quasi-experimental tracking and no-tracking studies conducted by Robert Slavin (1990) showed that when curriculum was held constant, tracking brought no advantage to any group, including high achievers. A similar meta-analysis conducted by C. Kulik and Kulik (1982) revealed a slight advantage for initial high achievers who studied in homogeneously grouped classes. Our study confirms Slavin's findings that initial high achievers' performance is not hurt if (a) curriculum is held constant and (b) the heterogeneity of initial achievement levels in the class expands. Our results also support the contention of Oakes et al. (1990), Slavin and Braddock (1993), and Wheelock (1992) that detracking will be effective if it occurs as a process of "leveling up"—that is, if all students receive the high-track curriculum.

Finally, this study does not support concerns that the performance of high achievers will decrease in heterogeneously grouped classes even if the high-track curriculum remains in place. The results of a matched-pairs *t* test did not reveal a statistically significant difference between the mean scores of initial high achievers before and after all students had studied the accelerated curriculum in detracked classes. In addition, providing universal accelerated mathematics instruction was associated with overall *increases* in advanced placement calculus scores among both all students and initial high achievers, even though more initial low achievers studied the course and took the exam.

Implications and Need for Further Research

At a time when all students are expected to meet high standards, this study of heterogeneous grouping coupled with accelerated mathematics provides valuable information for educators and scholars. Although it was based on data from the efforts of a single school, it provides an in-depth, unique analysis of the achievement of nearly 1,000 students. We believe that our results make an important contribution to the literature on the benefits of detracking when enriched, accelerated curricula are provided.

First, unlike previous studies of tracking, this study was longitudinal in design. We examined the effects of grouping practices on student performance and course taking, assessing several achievement measures for groups of students throughout their high school years and recording changes in the proportions of students studying advanced mathematics. In addition, unlike large studies of tracking that have based their findings on national achievement tests, the external evaluations of student performance used here, specifically the New York State regents and advanced placement calculus exams, measure the mastery of specific, taught curricula. Some researchers have suggested that the national achievement tests used in large studies of tracking

may not be the best measures of the effects of tracking because it is possible that both ability, defined as potential for growth, and actual knowledge are assessed by such tests (Kerckhoff, 1986; Slavin, 1990).

Second, our study provides insight into a question frequently asked by researchers of tracking: What is the cause of the higher performance associated with high-track classes? Kerckhoff (1986), reflecting on the results of his own study, suggested that the high-track advantage might result from differentiated curricula, better teachers in high-track classes, or classroom cultures. Our results suggest that the higher performance of students in high-track classes may very well stem from rigorous curricula and high expectations, not from the grouping practices in which students are sorted and selected. Strong teaching also plays a role. Previous research has demonstrated that high-track mathematics classes are more likely to be taught by mathematics majors who are both better qualified and more experienced (Oakes et al., 1990) and that low-track classes, in contrast, are more often taught by less able teachers whose skills appear to diminish over time (Finley, 1984). As a result of detracking, all students in this district had access to the same teachers.

Third, this study does not stand in isolation. Rather, it confirms the findings of other studies suggesting that an enriched, accelerated curriculum is more beneficial to at-risk learners and low-achieving students than a traditional remedial curriculum that slows down instruction (Bloom et al., 2001; Heubert & Hauser, 1999; Kifer et al., 1993; Mason et al., 1992; Peterson, 1989; White et al., 1996). Also, it challenges the commonly held assumption of mathematics teachers that students must master “the basics” before studying algebra (Grossman & Stodolsky, 1995; Loveless, 1994).

Further detracking in the study district in social studies has been associated with significant increases in student performance on the New York State global studies regent exams since detracking began in 10th-grade social studies. In addition, the district has also experienced significant increases in the number of students who earn New York State regents diplomas (Burris & Welner, 2005). Since the advent of universal acceleration in mathematics, the percentages of students studying physics (either regents physics or international baccalaureate physics) have increased, as have the percentages of students studying international baccalaureate chemistry, biology, or environmental systems. Furthermore, scores on the rigorous International Baccalaureate Mathematics Methods examination, administered in the 11th grade, have significantly increased in all student groups, including initial high achievers. Although such additional evidence goes beyond this study, it indicates a clear and consistent pattern of benefit to students as this school system continues to detrack and provide all students with the former “high-track” curriculum.

Finally, the widespread opposition to detracking, which partly rests on the belief that high achievers learn more in homogeneous high-track classes than in heterogeneous accelerated classes, is not supported by our findings. Although results will vary in districts at different resource levels, our study makes an important contribution to a debate that often focuses on the fear that high-achieving students will be “held back” if schools detrack.

All of the factors just outlined strengthen the generalizability of this study and illustrate the potential of increased achievement when detracking is combined with exposure of all students to the high-track curriculum and provision of support for struggling learners. Nevertheless, it is important that further research explore the essential components of this reform. The district that implemented the reform is a suburban district that has allocated generous resources in providing support to struggling students. Fifth-grade stanine scores in mathematics indicate that students in the district earn higher scores than the national average, and the proportion of low achievers in this study was proportionally lower than the number of average and high achievers. Would the reform work in a district with fewer resources and larger numbers of struggling students?

In addition, use of stanine scores precluded our ability to measure the effects of detracking on students at the highest and lowest levels of initial achievement—for example, the top 5% and the bottom 5%. Would these students benefit from detracked mathematics involving a “high-track” curriculum? We suggest that questions such as those just outlined be explored in future research.

An additional suggested area for further research is the specific contribution of heterogeneity to the success of mathematics acceleration. In terms of our first research question, which addressed the long-term benefits of acceleration, the contributions of heterogeneity and acceleration to later study of advanced mathematics classes were intermingled. To what extent did detracking contribute to the improvements observed here? Would continued heterogeneous grouping in mathematics increase or decrease advanced mathematics study at the high school level? Continued progress in detracking in the study district indicates that the combination of detracking and “high-track” curricula is the best means of increasing student achievement. Additional research on these topics will better inform policymakers, practitioners, and researchers interested in the effects of grouping on student achievement.

References

- Adelman, C. (1999). *Answers in the tool box: Academic intensity, attendance patterns and bachelor's degree attainment*. Washington, DC: U.S. Government Printing Office.
- Atanda, R. (1999). *Do gatekeeper courses expand education options?* (NCES Publication No. 1999303). Washington, DC: U.S. Department of Education.
- Bloom, H. S., Ham, S., Melton, L., & O'Brien, J. (2001). *Evaluating the accelerated schools approach: A look at early implementation and impacts on student achievement in eight elementary schools*. New York: Manpower Demonstration Research Corporation.
- Brewer, D. J., Rees, D. I., & Argys, L. M. (1995). Detracking America's schools: The reform without cost? *Phi Delta Kappan*, 77, 210–212, 214–215.
- Burris, C. C., & Welner, K. G. (2005). Closing the achievement gap by detracking. *Phi Delta Kappan*, 86, 594–598.
- Cogan, L. S., Schmidt, W. H., & Wiley, D. E. (2001). Who takes what mathematics and in which track? Using TIMSS to characterize U.S. students' eighth-grade mathematics learning opportunities. *Educational Evaluation and Policy Analysis*, 23, 323–341.

- Cook, T. D., & Campbell, D. T. (1979). *Quasi-experimentation: Design and analysis issues for field settings*. Boston: Houghton Mifflin.
- Darling-Hammond, L. (2003). Standards and assessments: Where we are and what we need. *Teachers College Record*. Retrieved March 5, 2003, from <http://www.tcrecord.org>
- Darling-Hammond, L., & Falk, B. (1997). Using standards and assessments to support student learning. *Phi Delta Kappan*, 79, 190–199.
- Epstein, J. L., & MacIver, D. J. (1992). *Opportunities to learn: Effects on eighth graders of curriculum offerings and instructional approaches* (Report No. 34). Washington, DC: Office of Educational Research and Improvement.
- Figlio, D. N., & Page, M. E. (2002). School choice and the distributional effects of ability tracking: Does separation increase inequality? *Journal of Urban Economics*, 51, 497–514.
- Finley, M. K. (1984). Teachers and tracking in a comprehensive high school. *Sociology of Education*, 57, 233–243.
- Finnan, C., & Swanson, J. D. (2000). *Accelerating the learning of all students*. Boulder, CO: Westview Press.
- Gamoran, A., & Weinstein, M. (1998). Differentiation and opportunity in restructured schools. *American Journal of Education*, 106, 385–415.
- Goycochea, B. B. (2000). College prep mathematics in secondary schools: Access denied. *Dissertations Abstracts International*, 61(4), 1333.
- Grossman, P. L., & Stodolsky, S. S. (1995). Content as context: The role of school subjects in secondary school teaching. *Educational Researcher*, 24(8), 5–11, 23.
- Hallinan, M. T. (1994). Tracking from theory to practice. *Sociology of Education*, 67, 79–91.
- Heubert, J. P. (2002). First do no harm: How the misuse of promotion and graduation tests hurts our neediest students. *Educational Leadership*, 60(4), 26–31.
- Heubert, J. P., & Hauser, R. M. (Eds.). (1999). *High stakes: Testing for tracking, promotion, and graduation*. Washington, DC: National Research Council.
- Horn, L., & Nunez, A. (2000). *Mapping the road to college: First-generation students' mathematics track, planning strategies and context of support* (NCES Publication No. 2000-153). Washington, DC: U.S. Department of Education.
- Hosmer, D. W., & Lemeshow, S. (2000). *Applied logistic regression* (2nd ed.). New York: Wiley.
- Kerckhoff, A. C. (1986). Effects of ability grouping in British secondary schools. *American Sociological Review*, 51, 842–858.
- Kifer, E. (1993). Opportunities, talents and participation. In L. Burstein (Ed.), *The IEA Study of Mathematics III: Student growth and classroom processes* (pp. 279–308). Oxford, England: Pergamon Press.
- Kifer, E., Wolfe, R. G., & Schmidt, W. H. (1993). Understanding patterns of student growth. In L. Burstein (Ed.), *The IEA Study of Mathematics III: Student growth and classroom processes* (pp. 101–127). Oxford, England: Pergamon Press.
- Kulik, C., & Kulik, J. A. (1982). Effects of ability grouping on secondary school students: A meta-analysis of evaluation findings. *American Educational Research Journal*, 19, 415–428.
- Kulik, J. A. (1992). *An analysis of the research on ability grouping: Historical and contemporary perspectives*. Storrs, CT: National Center of the Gifted and Talented.
- Levin, H. M. (1988). *Accelerated schools for at-risk students* (Report No. 142). New Brunswick, NJ: Center for Policy Research in Education, Rutgers University.
- Levin, H. M. (1997). Raising school productivity: An x-efficiency approach. *Economics of Education Review*, 16, 303–312.
- Loveless, T. (1994). The influence of subject matter on middle school tracking policies. *Research in Sociology in Education*, 10, 147–175.

- Loveless, T. (1998). *The tracking and ability grouping debate*. Retrieved January 1, 2005, from <http://www.edexcellence.net/foundation/publication/publication.cfm?id=127>
- Loveless, T. (1999). *The tracking wars: State reform meets school policy*. Washington, DC: Brookings Institution.
- Lucas, S. R. (1999). *Tracking inequality: Stratification and mobility in American high schools*. New York: Teachers College Press.
- Maggio, H. D. (1988). A study of identification procedures in an eighth grade acceleration program (New York). *Dissertations Abstracts International*, 49(9), 2524.
- Mason, D. A., Schroeter, D. D., Combs, R. K., & Washington, K. (1992). Assigning average-achieving eighth graders to advanced mathematics classes in an urban junior high. *Elementary School Journal*, 92, 587–599.
- McKnight, C. C. (1987). *The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. A national report on the Second International Mathematics Study*. Champaign, IL: Stipes.
- Menard, S. (1995). *Applied logistic regression analysis*. Thousand Oaks, CA: Sage.
- Mosteller, F., Light, R. J., & Sachs, J. A. (1996). Sustained inquiry in education: Lessons from skill grouping and class size. *Harvard Educational Review*, 66, 797–843.
- National Center for Education Statistics. (2002). *Digest of education statistics, 2001: Chapter 2. elementary and secondary education*. Washington, DC: Institute of Education Sciences, U.S. Department of Education.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- Natriello, G., & Pallas, A. M. (1999). *The development and impact of high stakes testing*. (ERIC Document Reproduction Service No. ED 443 871)
- New York State Education Department. (1984). *Action plan to improve elementary and secondary education*. Albany: University of the State of New York.
- New York State Education Department. (2001). *Regents examinations, regents competency tests and proficiency examinations: School administrator's manual 2001 edition*. Albany: University of the State of New York.
- New York State Education Department. (2004). *What is a similar school?* Albany: University of the State of New York.
- Oakes, J. (1982). The reproduction of inequity: The content of secondary school tracking. *Urban Review*, 14, 107–120.
- Oakes, J. (1986). Keeping track, Part 1: The policy and practice of curriculum inequality. *Phi Delta Kappan*, 68, 12–17.
- Oakes, J., Ormseth, T., Bell, R., & Camp, P. (1990). *Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science*. Santa Monica, CA: RAND.
- Pendergast, A. (Ed.). (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Research Council.
- Peterson, J. M. (1989). Remediation is no remedy. *Educational Leadership*, 46(6), 24–25.
- Riley, R. W. (1997). *Mathematics equals opportunity*. Washington, DC: U.S. Department of Education.
- Rose, H., & Betts, J. R. (2004). The effect of high school courses on earnings. *Review of Economics and Statistics*, 86, 497–513.
- Schmidt, W. H. (1993). The distribution of instructional time to mathematical content: One aspect of opportunity to learn. In L. Burstein (Ed.), *The IEA Study of Mathematics III: Student growth and classroom processes* (pp. 279–308). Oxford, England: Pergamon Press.
- Schmidt, W. H. (2004). A vision for mathematics. *Educational Leadership*, 61(5), 6–11.
- Schmidt, W. H., Houang, R., & Cogan, L. (2002). A coherent curriculum: The case of mathematics. *American Educator*. Retrieved September 15, 2003, from http://www.aft.org/american_educator/summer2002/curriculum.pdf

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- Schmidt, W. H., McKnight, C. C., Cogan, L. S., Jakwerth, P. M., & Houang, R. T. (1999). *Facing the consequences: Using TIMSS for a closer look at U.S. mathematics and science education*. Boston: Kluwer Academic.
- Singham, M. (2003). The achievement gap: Myths and reality. *Phi Delta Kappan*, 84, 586–591.
- Slavin, R. E. (1990). Achievement effects of ability grouping in secondary schools: A best-evidence synthesis. *Review of Educational Research*, 60, 471–507.
- Slavin, R. E., & Braddock, J. H., III. (1993). Ability grouping: On the wrong track. *College Board Review*, 168, 10–19.
- Stevenson, D. L., Schiller, K. S., & Schneider, B. (1994). Sequences of opportunities for learning. *Sociology of Education*, 67, 184–198.
- Wheelock, A. (1992). The case for untracking. *Educational Leadership*, 50(2), 6–10.
- White, P., Gamoran, A., Porter, A. C., & Smithson, J. (1996). Upgrading the high school mathematics curriculum: Mathematics course-taking patterns in seven high schools in California and New York. *Educational Evaluation and Policy Analysis*, 18, 285–307.

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